

**Physics 137B Section 1: Problem Set #4**  
**Due: 5PM Friday Feb 20 in 2<sup>nd</sup> floor LeConte-Birge Cross-Over**

**Suggested Reading for this Week:**

- Bransden and Joachain (B& J) section 8.2
- Hyperfine splitting is not discussed in B& J. The treatment in class is identical to Griffiths section 6.5

**Homework Problems:**

1. (Liboff Problem 13.15) Consider the quantum mechanical dumbbell from problem 5 of Problem set 3. If both ends are equally charged, the rotating dumbbell has angular momentum  $\vec{L}$  and the corresponding magnetic dipole moment is

$$\vec{\mu} = \frac{e}{2mc} \vec{L}$$

where  $e$  is the net charge of the dipole. The interaction energy between this magnetic dipole and a constant uniform magnetic field  $\vec{B}$  is

$$H' = -\vec{\mu} \cdot \vec{B}$$

- (a) If  $\vec{B}$  points in the  $z$  direction, show that  $H'$  separates the  $(2\ell + 1)$ -fold degenerate  $E_\ell$  energies of the rotating dipole
- (b) Apply these results to find the splitting of the  $P$  states of the hydrogen atom (neglecting for now the spin-orbit couple).

We will return to this phenomenon, *the Zeeman effect*, later this semester.

2. (Liboff Problem 12.6) In quantum mechanics, when one says that the vector  $\vec{J}$  is conserved, one means that for any state the system is in, the expectation values of the 3 components of  $\vec{J}$  are constant. This follows if these 3 operators all commute with  $H$ . Show that for the hydrogen atom with spin-orbit coupling  $\vec{L}$  and  $\vec{S}$  are not conserved but  $\vec{J}$  is.

3. (Griffiths Problem 6.26) By appropriate modification of the hydrogen formula, determine the hyperfine splitting in the ground state of
- (a) Muonic hydrogen (in which a muon - same charge and  $g$ -factor as the electron but 207 times the mass - substitutes for the electron)
  - (b) positronium (in which a positron - same mass and  $g$ -factor as the electron but opposite charge - substitutes for the proton)
  - (c) Muonium (in which the antimuon - same mass and  $g$ -factor as a muon but opposite charge - substitutes for the proton and the muon substitutes for the electron)

Hint: Don't forget to use the reduced mass when necessary. Note: The answer you get for positronium is quite far from the experimental value of  $8.41 \times 10^{-4}$  eV. This discrepancy is due to pair annihilation ( $e^+e^- \rightarrow \gamma\gamma$ )

4. (Griffiths Problem 6.33) Calculate the wavelength, in cm, of the photon emitted under a hyperfine transition in the ground state ( $n = 1$ ) of deuterium. Deuterium is "heavy" hydrogen with an extra neutron in the nucleus. The proton and neutron bind together to form a deuteron with spin 1 and magnetic moment

$$\vec{\mu}_d = \frac{g_d e}{2m_d} \vec{S}_e$$

The deuteron  $g$ -factor is 1.71.